Selection index - solutions to exercises

1. Performance testing, 2 registrations:

(α(2 rec)) (α)

$$\frac{1+(2-1)0.4}{2}\,b = 1\times 0.3$$

which gives $b = 0.4286$ $\qquad r_{TI} = \sqrt{b\times a_{i\alpha}} = \sqrt{0.7241\times 0.5} = 0.65$

7 progenies (HS):

(7 prog (HS)) (α)

$$\frac{1+(7-1)0.25\times 0.3}{7}\quad b = 0.5\times 0.3$$

which gives $b = 0.7241$ $\qquad r_{TI} = \sqrt{0.7241\times 0.5} = 0.60$

Conclusion: Performance testing gives somewhat higher accuracy.

2. a) The ram itself = X_1 (1 reg.) = α

$$\frac{\frac{1+(n_1-1)r}{n_1}+(p_1-1)\times a_{11}\times h^2}{p_1}\times \sigma_X^2 \times b \quad = \quad a_{1\alpha}\times h^2 \times \sigma_X^2$$

When $n_1 = p_1 = 1$ and $a_{1\alpha} = 1$ we get

$1 \cong b = 1 \cong h^2$

$b = h^2 = 0.60$

$$r_{TI} \quad = \quad \sqrt{\sum_{i=1}^{n}\frac{b_i a_{i\alpha}\times h^2 \times \sigma_X^2}{\sigma_T^2}} \quad = \quad \sqrt{b_i \times a_{i\alpha}} \quad = \quad \sqrt{b} \quad = \quad 0.77$$

b) The ram itself = α $\qquad a_{1\alpha} = \frac{1}{2}$

$\uparrow$

10 progenies = X_1 (1 reg.)

$$n_1 = 1$$
$$P_1 = 10$$
$$a_{11} = a_{HS} = \frac{1}{4}$$

$$\frac{\frac{1+(1-1)r}{1}+(10-1)\times\frac{1}{4}\times 0.60}{10}\times b \quad = \quad \frac{1}{2}\times 0.60$$

$$\frac{\frac{1+9}{1}\times\frac{1}{4}\times 0.6}{10}\times b \quad = \quad 0.30 \qquad \text{which gives } b = 1.28$$

$$r_{TI} \quad = \quad \sqrt{b_i \times a_{i\alpha}} \quad = \quad \sqrt{1.28\times\frac{1}{2}} \quad = \quad 0.80$$

For such a high heritability, information on the ram itself gives almost equally accurate estimates of the breeding value as information on 10 progenies.

3. a) Dam of the cow = X_2 (1 rec.)

$$\downarrow$$

Cow itself = α = X_1 (1 rec.)

$$\hat{T} = I = b_1X_1 + b_2X_2$$

when $a_{12} = 0.5$; $a_{1\alpha} = 1$; $a_{2\alpha} = 0.5$

$h^2 = 0.20$ we get

$$\begin{array}{ccc} & (\alpha) & (\mathrm{Dam}) & & & (\alpha) \\ \begin{array}{c}(\alpha)\\(\mathrm{Dam})\end{array} & \multicolumn{2}{c}{\begin{bmatrix} 1 & 0.5\times 0.20 \\ 0.5\times 0.20 & 1 \end{bmatrix}} & \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} & = & \begin{bmatrix} 1\times 0.20 \\ 0.5\times 0.20 \end{bmatrix} \end{array}$$

$$\begin{cases} b_1 \times 1 + b_2 \times 0.10 = 0.20 \\ b_1 \times 0.10 + b_2 \times 1 = 0.10 \end{cases}$$

$$\begin{cases} b_1 \times 0.1\ b_2 = 0.20 \\ 0.1\ b_1 + b_2 = 0.10 \end{cases} \Rightarrow \begin{cases} b_1 = 0.192 \\ b_2 = 0.081 \end{cases}$$

b_i is put into the index equation

$I = 0.192X_1 + 0.081X_2$

b) $$r_{TI} = \sqrt{\sum_{i=1}^{n} b_i a_{i\alpha}} = \sqrt{b_1a_{1\alpha} + b_2a_{2\alpha}} = \sqrt{0.192\times 1 + 0.081\times 0.5} = \sqrt{0.232} = 0.482$$

c) $I = b_1X_1$

$b_1 \times 1 = 1 \times \cong h^2$

$b_1 = h^2 = 0.20$

$$r_{TI} = \sqrt{0.20 \times 1} = 0.447$$

For a medium high heritability as this one, we loose comparatively little if we do not include information on the dam of the cow r_{TI} decreases from 0.482 to 0.447.

4. a)

Dam of the cow = X_2 (1 rec.)

Maternal sister
of the cow = X_3 (1 rec.) $\longrightarrow$ The cow itself = α = X_1 (1 rec.)

$I = b_1X_1 + b_2X_2 + b_3X_3$

$$\begin{array}{cc}
 & \begin{array}{ccc} (\alpha) & (\text{Dam}) & (\text{1HS}) \end{array} \\
\begin{array}{c} (\alpha) \\ (\text{Dam}) \\ (\text{1HS}) \end{array} & \begin{bmatrix} 1 & 0.5\times0.20 & 0.25\times0.20 \\ 0.5\times0.20 & 1 & 0.5\times0.20 \\ 0.25\times0.20 & 0.5\times0.20 & 1 \end{bmatrix}
\end{array}
\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} =
\begin{array}{c} (\alpha) \\ \begin{bmatrix} 1\times0.20 \\ 0.5\times0.20 \\ 0.25\times0.20 \end{bmatrix} \end{array}$$

b)

$$r_{TI} = \sqrt{\sum_{i=1}^{n} b_i a_{i\alpha}} = \sqrt{0.191\times1 + 0.078\times0.5 + 0.033\times0.25} = \sqrt{0.2387} = 0.49$$

This is the highest r_{TI} value. However it is only slightly higher than in exercise 3 which shows that a record on a maternal sister only contributes to a small extent to the accuracy of the breeding value estimation.

5. a) $h_1^2 = \dfrac{88.2}{187.7} = 0.47$

$h_2^2 = \dfrac{5.2}{15.2} = 0.34$

$r_p = \dfrac{\sigma_{p_1p_2}}{\sqrt{\sigma^2_{p_1}\sigma^2_{p_2}}}$

$\sigma_{p_1p_2} = -0.05\times\sqrt{187.7\times 15.2} = -2.67$

$r_g = \dfrac{\sigma_{A_1A_2}}{\sqrt{\sigma^2_{A_1}\times\sigma^2_{A_2}}}$

$\sigma_{A_1A_2} = -0.54\times\sqrt{88.2\times 5.2} = -11.56$

b) I = $b_1X_1 + b_2X_2$

T = $v_1T_1 + v_2T_2$

$\underline{P}\ \underline{b} = \underline{G}\ \underline{v}$

$$\begin{array}{c c} & \begin{array}{cc} (1) & (2) \end{array} \\ \begin{array}{c} (1) \\ (2) \end{array} & \begin{bmatrix} 1\times\sigma^2_{X_1} & \sigma_{X_1X_2} + 0 \\ \sigma_{X_1X_2} + 0 & 1\times\sigma^2_{X_2} \end{bmatrix} \end{array} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{array}{c} \begin{array}{cc} (1) & (2) \end{array} \\ \begin{bmatrix} 1\times\sigma^2_{A_1} & 1\times\sigma_{A_1A_2} \\ 1\times\sigma_{A_1A_2} & 1\times\sigma^2_{A_2} \end{bmatrix} \end{array} \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} 187.7 & -2.67 \\ -2.67 & 15.2 \end{bmatrix}\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 88.2 & -11.56 \\ -11.56 & 5.2 \end{bmatrix}\begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

c) T = T_2

$$\begin{array}{c c} & \begin{array}{cc} (1) & (2) \end{array} \\ \begin{array}{c} (1) \\ (2) \end{array} & \begin{bmatrix} 187.7 & -2.67 \\ -2.67 & 15.2 \end{bmatrix} \end{array} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{array}{c} (2) \\ \begin{bmatrix} -11.56 \\ 5.2 \end{bmatrix} \end{array}$$